Sensing Lena—Massively Distributed Compression of Sensor Images

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Acknowledgements

- Ron Dabora (PhD candidate at Cornell), An-swol Hu (PhD candidate at Cornell), Megan Owen (PhD candidate at Cornell), Christina Peraki (PhD candidate at Cornell), João Barros (PhD candidate at TU München, visitor at Cornell).


- NSF, for awards CCR-0238271(CAREER), CCR-0330059(SENSORS); ONR, for an infrastructure grant to set up a sensor testbed; Professor Joachim Hagenauer (TU München).
Outline

• The Sensor Broadcast Problem
  – Problem Statement, Applications/Relevance, Main Challenges.

• Performance Measures for Broadcast Protocols
  – Achievability of $R_X(D/n)$: Sampling + Scalar Quantization.
  – Non-Achievability of $R_X(D/n)$: Scalar Quantization + Entropy Coding.
  – Maybe-Achievability of $R_X(D/n)$: Transform Coding.

• Wavelets and Sensor Broadcast
  – Basis Functions with Compact Support and Local Communication.
  – A Wavelet-Based Protocol.
  – Achievability of $R_X(D/n)$ using Wavelets.

• Summary and Conclusions

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The Sensor Broadcast Problem—Formulation

Each node broadcasts some local information from a 2-D random field, all nodes must form a rate-constrained estimate of the entire field:

Why do we care? Because of its extremal properties, and because we can use it as a building block for other more complex protocols...

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Distributed Software Radio—An Application for Broadcast

If all nodes can be “sync’d” to the same data bits, they can all cooperate to generate a strong information bearing signal at a far receiver...


The Infamous Borg

How can the drones maintain a collective, global memory?

The Sensor Broadcast Problem—Is There Any Hope?

Does the network have enough capacity to support broadcast?

For a network with $n$ nodes, with optimal routing, optimal power control, and optimal transmission schedules, the total transport capacity scales like $O(\sqrt{n})$—i.e., $O(1/\sqrt{n}) \to 0$ bits/node.


The answer is maybe:

- With independent encoders, to keep $\bar{D} = \frac{1}{n} \sum_{i,j} d(X_{ij}, \hat{X}_{ij})$ bounded, we need $O(\log n) \to \infty$ bits/node... i.e., the network chokes!

- But independent encoders ignore the fact that as $n \to \infty$, the data becomes increasingly correlated...

The Sensor Broadcast Problem—Previous Work

- For any 2D bandlimited sensor field \( \mathbf{X} = [X_1 \ldots X_n] \), \( R_X(D/n) \) grows as \( \Theta(\log(n/D)) \). Hence, fixing \( D/n \), it must be possible for the network to compress sensor fields to a constant number of bits, independent of \( n \).


- Quote (sic): “we show that as the density (of the network) increases to infinity, the total number of bits required to attain a given quality also increases to infinity under any compression scheme”.


So, what is the case? For fixed \( D/n \), is it possible or not for the sensors to encode bandlimited images into a number of bits that is independent of \( n \)?

**In this talk we show the answer is yes, this is possible.**
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**Achievability of** $R_X(D/n)$: **Sampling + Scalar Quantization**

Consider $f \in \mathcal{L}^2(\mathbb{R})$, with Fourier transform $F(\omega)$ and bandwidth $\Omega$:

- **Uniform sampling:**
  - Uniformly sample $f$ at rate $2\Omega$, over duration $T$.
  - Scalar quantize + entropy code each sample independently.
  - At distortion $D/n$, the total traffic generated is upper bounded by
    \[
    \sum_{k=1}^{2\Omega T} \left( 0.5 \log(\sigma^2 n/D) + 0.255 \right) = c_1 \log(n/D) + c_2 = \Theta(R_X(D/n))
    \]

- **Random sampling:**
  - Let $P(\omega) = \sum_i f(t_i)e^{-i\omega t_i}/\lambda$, $t_i \sim \text{Poisson}(\lambda)$. If $\lambda > 2\Omega$, $P(\omega) \approx F(\omega)$.
  - Randomly sample $f$ at fixed rate $\lambda > 2\Omega$. **Same rate of growth.**

So, $R_X(D/n)$ is indeed achievable. This system though complies only with the “letter”, but not with the “spirit” of the law... (Emre Telatar dixit).

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Non-achievability: Scalar Quantization + Entropy Coding

A most interesting observation:

Can choose $T$ so that the continuous valued stopping time

$$t_0 = \min_{t \in \mathbb{R}} \left\{ X(t) \geq T \right\}$$

can be estimated from the quantized samples $\hat{X}(\ell_1)\ldots\hat{X}(\ell_N)$.

Hence, $\mathcal{H}(\hat{X}(\ell_1)\ldots\hat{X}(\ell_N)) \geq \mathcal{H}(t_0) \rightarrow \infty$, for any scalar quantizer.


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**Maybe-Achievability of** $R_x(D/n)$: Transform Coding

If we look at the proof of $R_x(D/n) = \Theta(\log(n/D))$, we recognize a key step: most eigenvalues of $R_{xx}$ vanish, its null space increases with $n$.

**Hence, if we could decorrelate, most coefficients would have vanishing variance**—potential for big (enough?...) compression gains!

- For $X \sim N(0, R_{xx})$, optimal processing is given by the KLT $Y = U \cdot X$:
  - $R_{xx} = U \Lambda U^\top$, $\Lambda = \text{diag}(\lambda_1, \ldots, \lambda_n)$.
  - $U = [\phi_1 \ldots \phi_n]$, basis of eigenvectors of $R_{xx}$.

- Since $R_{xx}$ is doubly-Toeplitz, as $n \to \infty$, $U$ becomes a 2D-DFT matrix.


So, we would like to decorrelate data to implement broadcast efficiently, but to decorrelate we need to solve a broadcast problem first...
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Compact Support and Local Communication

Can we approximate the optimal KLT with a suboptimal transform that is “less demanding” in terms of communication requirements?

Yes: wavelets with compact support.

Plus, as a bonus... wavelet-based image coding is a very well understood problem, so we can recycle all that knowledge in this context.
A Wavelet-based Sensor Broadcast Protocol


Achievability of $R_X(D/n)$ using Wavelets—Main Ideas

Outputs of the highpass branch in the filter bank will contain non-negligible energy only for inputs sampled at a rate close to critical...

- To deal with non-ideal filters and A/D noise, add a hard threshold.
- Good (best?) filters with given length: Daubechies’ maxflat family.
- Communication intensive only within area $= \Theta(1/$ spatial bandwidth$)$.
- Robust to lack of knowledge on field statistics: oversample + compress.


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IEEE ICIP—9/15/03.
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Our main contribution:

Elements of a system capable of compressing down to a finite-length bit stream all the measurements captured by a dense sensor network.

Key: while $\mathcal{H}(\hat{X}_1...\hat{X}_n)$ may grow unbound, there are transforms $\mathbf{f}$ such that:
(a) $\mathcal{H}(\mathbf{f}(\hat{X}_1...\hat{X}_n))$ does not, and (b) $\mathbf{f}$ can be computed within the network.

- $R_X(D/n)$ is indeed achievable with distributed processing of all samples, without turning off sensors—letter and spirit!

- After the dust settles, sensor broadcast does work.

Currently working on implementing a broadcast algorithm, for use in a real sensor network prototype—work with http://www.iwtwireless.com/.

Ladran Sancho, señal que cabalgamos...”
— Don Quixote de la Mancha

Main Corollary...

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