The EGC region is given by:

$$
\begin{aligned}
R_1 &> I(X;\hat{X}_1) && (1)\\
R_2 &> I(X;\hat{X}_2) && (2)\\
R_1+R_2 &> I(X;\hat{X}_0\hat{X}_1\hat{X}_2)+I(\hat{X}_1;\hat{X}_2) && (3)
\end{aligned}
$$

The conditions from the theorem are

$$
\begin{aligned}
R_1' &> I(X;\hat{X}_1) && (4)\\
R_2' &> I(X;\hat{X}_2) && (5)\\
R_1'+R_2'-I(\hat{X}_1;\hat{X}_2) &> I(X;\hat{X}_1\hat{X}_2) && (6)\\
\Delta_1+\Delta_2 &> I(X;\hat{X}_0|\hat{X}_1\hat{X}_2) && (7)
\end{aligned}
$$

We need to show that for any pair of rates (R_1,R_2) that satisfy $(1)-(3)$, we can obtain $(R_1',R_2',\Delta_1,\Delta_2)$ that satisfy $R_i=R_i'+\Delta_i$ and the conditions $(4)-(7)$.

Consider a rate pair (R_1,R_2) that satisfy the conditions of the EGC region. Let

$$
\begin{aligned}
\delta_1 &= R_1-I(X;\hat{X}_1)\\
\delta_2 &= R_2-I(X;\hat{X}_2)
\end{aligned}
$$

Substituting these in condition (3), we get (after a bunch of chain rules)

$$
\begin{aligned}
\delta_1+\delta_2 &> \underbrace{I(X;\hat{X}_0,\hat{X}_1\hat{X}_2)-I(X;\hat{X}_2)}+I(\hat{X}_1;\hat{X}_2)-I(X;\hat{X}_1)\\
&= \underbrace{I(X;\hat{X}_0\hat{X}_1|\hat{X}_2)}+I(\hat{X}_1;\hat{X}_2)-I(X;\hat{X}_1)\\
&= I(X;\hat{X}_0|\hat{X}_1,\hat{X}_2)+\underbrace{I(X;\hat{X}_1|\hat{X}_2)+I(\hat{X}_1;\hat{X}_2)}-I(X;\hat{X}_1)\\
&= I(X;\hat{X}_0|\hat{X}_1,\hat{X}_2)+\underbrace{I(\hat{X}_1;X\hat{X}_2)-I(\hat{X}_1;X)}\\
&= I(X;\hat{X}_0|\hat{X}_1\hat{X}_2)+I(\hat{X}_1;\hat{X}_2|X)
\end{aligned}
$$

We can now split the sum $\delta_1+\delta_2$ as $\epsilon_1+\epsilon_2$ and $\Delta_1+\Delta_2$ such that

$$
\begin{aligned}
\Delta_1+\Delta_2 &> I(X;\hat{X}_0|\hat{X}_1\hat{X}_2) && (8)\\
\epsilon_1+\epsilon_2 &> I(\hat{X}_1;\hat{X}_2|X) && (9)
\end{aligned}
$$

Now, if we write $R_i'=R_i+\epsilon_i$, and substitute into (9), we get (after more chain rules)

$$
\begin{aligned}
R_1'+R_2'-I(\hat{X}_1;\hat{X}_2) &> \underbrace{I(\hat{X}_1;\hat{X}_2|X)+I(X;\hat{X}_1)}+I(X;\hat{X}_2)-I(\hat{X}_1;\hat{X}_2)\\
&= \underbrace{I(\hat{X}_1;X\hat{X}_2)-I(\hat{X}_1;\hat{X}_2)}+I(X;\hat{X}_2)\\
&= I(X;\hat{X}_1|\hat{X}_2)+I(X;\hat{X}_2)\\
&= I(X;\hat{X}_1\hat{X}_2)
\end{aligned}
$$